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ORIGINAL ARTICLE

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Abstract With *p* and *q* each standing for a familiar event, a disjunctive statement, "either *p* or *q*", seems quite different from its material conditional, "if not *p* then *q*". The notions of sufficiency and necessity seem specific to conditional statements. It is surprising, however, to find that perceived sufficiency and necessity affect disjunctive reasoning in the way they affect conditional reasoning. With B and C each standing for a category name, a universal statement, "all B are C", seems stronger than its logically equivalent conditional statement, "if B then C". However, the effects of perceived sufficiency or necessity were found to be as pronounced in conditional reasoning as in syllogistic reasoning. Furthermore, two experiments also showed that (a) MP (modus ponens)-comparable disjunctive reasoning was as difficult as MT (modus tollens)-comparable disjunctive reasoning, and that (b) MT-comparable syllogisms were easier to solve than MT problems in conditional reasoning.

Introduction

Together with conditional reasoning, disjunctive and syllogistic reasoning could be the three most known types of deductive reasoning in psychology (e.g., Evans, Newstead, & Byrne, 1993; Manktelow, 1999). If a successive-conditionalization approach can be used to understand conditional reasoning (e.g., Chou, 2009; Klauer, Beller, & Hutter, 2010; Liu, 2003; Liu, Lo, & Wu, 1996; Matarazzo & Baldassarre, 2008; Schroyens, 2009), it could also be used to understand disjunctive and syllogistic reasoning, because this approach applies to any type of deductive reasoning. This is the main objective of the present study.

With *p* (e.g., George moves to a new house) and *q* (he adds some furniture) representing real world events about which people have some knowledge, consider the case of Modus Ponens (MP):

> If "George moves to a new house" (*p*), then "he adds some furniture" (*q*).
>
> George moves to a new house (*p*).
> Therefore, he adds some furniture (*q*).

In the successive-conditionalization approach, reasoners first compute the probability of the conclusion given the second premise, i.e., P (*q* | *p*), based on the reduced problem in which the conditional premise is deleted (e.g., "Given that George moves to a new house, how probable is that he adds some furniture?"). The *first conditionalization* is used to stand for P (conclusion | second premise). Because *p* and *q* represent real world events about which participants have some knowledge, they naturally calculate this conditional probability, based on their world knowledge. This is the reason why the first conditionalization is called knowledge-based reasoning.

Reasoners then take into consideration that the first premise is true in attempting to solve the complete problem (e.g., "If George moves to a new house, then he adds some furniture. Given that George moves to a new house, how

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probable is that he adds some furniture?"). Computing P (conclusion | second premise) under the conditional-premise assumption stands for the *second conditionalization.* Thus, the second conditionalization for MP is represented by P (*p* therefore *q* | if *p* then *q*).

In computing the second conditionalization, there are several possibilities. First, as George (1995) pointed out, some participants would follow the instructions of assuming the conditional premise to be true by taking *p* to be sufficient for *q*. It was shown in an experimental study (Liu, 2010a) that the majority of college students could comprehend the conditional to mean that *p* is sufficient for *q*, when a precaution is exercised for participants to detach from reality. In this case, P (*p* therefore *q* | if *p* then *q*) = 1. Second, some participants would be unable to detach from reality completely, interpreting P (if *p* then *q*) to be equal to P (*q* | *p*). This case is similar to the result of the Ramsey test (1931), or the case in which participants are to judge the believability or probability of conditionals (e.g., Evans, Handley, Over, 2003; Oberauer and Wilhelm, 2003; Over, Hadjichristidis, Evans, Handlely, & Sloman, 2007). Third, many participants would try to comply with the instructions of assuming the conditional premise to be true, while they could have some doubt about its truth. This case would result in their inability to perform the second conditionalization completely, producing P (*q* | *p*) < P (*p* therefore *q* | if *p* then *q*) < 1. The extent to which the first conditionalization is enhanced by the second is referred to as the assumption-based or form-based reasoning. The distinction between the first and second conditionalization is only theoretical. In actuality, reasoners could process the two premises simultaneously.

In brief, the successive-conditionalization approach predicts that MP probability will range between P (*q* | *p*) and 1, according to the extent to which the first conditionalization is enhanced by the second. This is because, in performing the second conditionalization, P (*p* therefore *q* | if *p* then *q*), reasoners are unable to detect *p* to be completely sufficient for *q* in "if *p* then *q*" by detaching from reality. The same applies to the other valid inference, Modus Tollens (MT). For the two fallacies, denial of the antecedent (DA) and affirmation of the consequent (AC), it is predicted that the second conditionalization will add nothing, so that they equal P (conclusion | second premise), unless the conditional premise is interpreted as a biconditional.

The prediction that the first conditionalization is an important component of MP, MT, DA, and AC is well established (e.g., Liu, 2003; Liu et al., 1996). These results are consistent with the observations that MP and MT are affected by perceived sufficiency (e.g., Byrne, 1989; Cummins, Lubart, Alksnis, & Rist, 1991; Rumain, Cornell, & Braine, 1983; Staudenmayer, 1975; Thompson, 1994) and that DA and AC are affected by perceived necessity (e.g., Bucci, 1978, Experiment 2; Byrne, 1989; Cummins et al., 1991; Rumain et al., 1983; Markovits, 1984; Staudenmayer, 1975; Thompson, 1994, 1995). Another prediction that some variables affect the second conditionalization without affecting the first conditionalization is also well established. Problem content is such a variable. Thus, MP responses of abstract content were significantly higher than MP responses of thematic content, although the first conditionalization of the former was the same as that of the latter (Liu, 2003). This is because it is easier for participants to detach from reality in the case of abstract content than in the case of thematic content in performing the second conditionalization.

The successive-conditionalization approach is feasible, because computing the probability of the conclusion given the two premises is shown to be equivalent to performing the successive conditionalization on the two premises (e.g., Jeffrey, 1981). The formula for successive conditionalization can also be derived from the generalized Bayes' formula (e.g., Cox, 1961). Oaksford, Chater, and Larkin (2000) predict that P (conclusion | second premise) holds not only for DA and AC, but also for MP and MT. Oaksford and Chater (2007) considered that the conditional premise is not disregarded in their predictions, but reasoners are assumed to conditionalize on it. Thus, their approach differs from the present approach in which the reduced problems are separately administered from the complete problems. Although there is another probabilistic approach (e.g., Pfeifer & Kleiter, 2007) based on mental probability logic (e.g., Pfeifer & Kleiter, 2005), because participants were instructed to assume the premises to be true in the present study as in most studies, this probabilistic approach does not apply to the present study.

The main difference between the present approach and current dual processing theories (see Evans, 2008 for review) could be that both knowledge-based and assumption-based reasoning of the present approach are experimentally identified, while the two processes in many other dual processing theories are often estimated through parameters. In the present approach, as in any other deductive reasoning task, participants are instructed to assume the premises true. Nevertheless, if the probability of conditionals is basically the subjective conditional probability (conditional probability hypothesis), it may be doubtful whether participants could be instructed to assume the conditional-statement premise true. It is necessary, therefore, to consider a dispute on what the probability of conditionals is as follows.

There has been a continuing dispute on what the probability of conditionals is between the suppositional account (e.g., Evans, Over, & Handley, 2005; Evans & Over, 2010) and the mental model account (e.g., Barrouillet, Gauffroy, & Lecas, 2008; Byrne & Johnson-Laird, 2010). According

to the suppositional account (Evans & Over, 2004; Evans et al., 2005), people suppose that the if-clause is true, and think about the consequences. The probability of the conditional is derived in a process that is sensitive to the relative frequency of cases with *p* and *q* versus cases with *p* and *not q*. The probability of the conditional is then the subjective conditional probability. The mental model account (e.g., Byrne & Johnson-Laird, 2009), on the other hand, predicts the probability of 3/4, because a conditional generally holds in three out of the four equally likely outcomes.

It is argued (Liu, 2010b) that overwhelming evidence in support of the conditional probability hypothesis is obtained, because people are asked to evaluate the conditional in terms of probability and it is extremely difficult to detach from reality in rating the conditional. When precautions were exercised to avoid these difficulties, the majority of participants comprehend the conditional to mean that the antecedent is sufficient for the consequent. The suppositional account that is embedded in a dual-process framework could explain this finding, because the analytic system could operate on decontextualized task representations (e.g., Stanovich, 1999) and participants could detach from reality in the judgment of conditionals. The mental model account is also consistent with this finding, because Johson-Laird and Byrne (2002, p. 649) basically assume that *p* is sufficient for *q*. It may be concluded that both accounts might not be as opposed as they first appear.

Experiment 1: disjunctive versus conditional reasoning

Although disjunctive reasoning has rarely been studied in the past decade, recent papers have frequently referred to disjunctive statements in connection with the material interpretation of conditionals (e.g., Evans et al., 2003; Oberauer & Wilhelm, 2003; Over et al., 2007). The relative difficulty of conditional and disjunctive reasoning has been investigated. Johnson-Laird, Byrne, and Schaeken (1992) claimed that disjunctives are harder to reason with than conditionals, based on the idea that disjunctives have a more complex mental model. Their own research with a conclusion production task supports their claim that disjunctives are indeed more difficult than conditionals. Although Klauer and Oberauer's study (1995) generally supports the Johnson-Laird et al. claim, other studies failed to support it. Roberge (1974, 1978) found no difference in difficulty. Roberge and Antonak (1979) found differences that vary in both direction and magnitude as a function of the content used.

In the material interpretation of conditionals, the disjunctive "Either *p* or *q*" is equivalent to the conditional "If not *p* then *q*". In the successive-conditionalization framework, both types of reasoning have the identical first conditionalization, P (conclusion | second premise). Thus, it is possible to construct four argument forms for disjunctive reasoning that are comparable to the four forms of conditional reasoning (MP, MT, DA, and AC) in Table 1, as follows (The second premise and the conclusion are stated in one-sentence form):

Table 1 Conditional versus comparable disjunctive argument forms

Conditional		Disjunctive	
MP:	If not p then q Not p, therefore q	MP′:	Either p or q Not p, therefore q
MT:	If not p then q Not q, therefore p	MT′:	Either p or q Not q, therefore p
DA:	If not p then q p, therefore not q	DA′:	Either p or q p, therefore not q
AC:	If not p then q q, therefore not p	AC′:	Either p or q q, therefore not p

MP modus ponens, *MT* modus tollens, *MP′* comparable to MP, *MT′* comparable to MT, *DA* denial of the antecedent, *AC* affirmation of the consequent, *DA′* comparable to DA, *AC′* comparable to AC

The first two disjunctive argument forms that are comparable to conditional MP and MT are known as the denial inferences. The last two argument forms of disjunctive reasoning are known as the affirmation inferences, which are comparable to conditional DA and AC forms. Both denial inferences are valid, while both affirmation inferences are invalid. However, both affirmation inferences are valid in the exclusive reading of disjunctives (*p* or *q* but not both). This is in parallel to a conditional versus biconditional distinction, because in biconditional reading DA and AC inferences are valid.

The main purpose of Experiment 1 was to test whether the material interpretation of conditionals could explain disjunctive and conditional reasoning processes. In the successive-conditionalization approach, it is possible to separate the effect of variables that affect the first conditionalization (i.e., knowledge-based reasoning) from the effect of variables that affect assumption-based reasoning. In this framework, the material implication hypothesis predicts that the first conditionalization is not only a significant component of the second conditionalization, but the second conditionalization should be identical between conditional and disjunctive reasoning. On the other hand, the present approach predicts that the second conditionalization should differ between conditional and disjunctive reasoning, although the first conditionalization could be a significant component of the second conditionalization in both conditional and disjunctive reasoning.

More specifically, both approaches predict that perceived sufficiency and necessity affect disjunctive reasoning responses by affecting the first conditionalization as they affect conditional reasoning responses by affecting the

first conditionalization. However, the present approach predicts that disjunctive reasoning differs from conditional reasoning with respect to the second conditionalization. This is because the second conditionalization will be affected by differences in the surface features of the major premises. In performing the second conditionalization in MP inferences, reasoners attempt to compare "given not *p*, therefore *q*" to "if not *p* then *q*". It is easy for reasoners to detect their relationship, because "if not *p* then *q*" states that not *p* is sufficient for *q*. On the other hand, in performing the second conditionalization in MP-comparable denial inferences, reasoners attempt to compare "given not *p*, therefore *q*" to "either *p* or *q*". It is not easy to see their relationship immediately. Therefore, in the successive-conditionalization approach the assumption-based reasoning in MP inferences is predicted to be easier than the assumption-based reasoning in MP-comparable denial inferences. On the other hand, the assumption-based reasoning in MT-comparable denial inferences are predicted to be as easy as the assumption-based reasoning in MP-comparable denial inferences in disjunctive reasoning, because both are rather symmetrical in performing the second conditionalization.

Method

In conditional reasoning, perceived sufficiency is known to affect MP/MT responses by affecting the first conditionalization, while perceived necessity to affect DA/AC responses directly (e.g., Liu, 2003). The first objective was, therefore, to find whether perceived sufficiency is a significant variable that affects the denial inferences and whether perceived necessity is a significant variable that affects the affirmation inferences in disjunctive reasoning. The second objective of conducting this experiment was to assess how the difference in the major premises (that are logically identical in the material interpretation) between disjunctive and conditional reasoning produces differences in these two types of reasoning by affecting their assumption-based reasoning.

Because four disjunctive argument forms are comparable to the four conditional argument forms in the material interpretation of conditionals, it should be possible to generate the reduced problems for disjunctive reasoning as for conditional reasoning. Thus, for instance, the reduced disjunctive reasoning problems that are comparable to reduced MP problems take the following form: given not *p*, therefore *q* (see Table 1). As is apparent from Table 1, each reduced disjunctive reasoning problem is identical to its comparable reduced conditional reasoning problem.

In order to manipulate perceived sufficiency and necessity, the next task is to find a sufficient number of such items characterized by high sufficiency, medium sufficiency, and low sufficiency, while all the items are characterized by low necessity. If such items could be found, it is possible to find a sufficient number of items characterized by high necessity, medium necessity, and low necessity while all the items are characterized by low sufficiency by reversing the antecedent and consequent clauses of each conditional statement (e.g., Cummins, 1995; Thompson, 1994). To take an example ("If not *p*, then *q*"), suppose that "given not *p*, therefore *q*" is characterized by high sufficiency and low necessity, then "given *q*, therefore not *p*" should be characterized by high necessity and low sufficiency.

It turned out that it is impossible to find high sufficiency items characterized by low necessity. For example, given that a person is not male (=not *p*) the probability of this person being female (=*q*) should be very large, which measures perceived sufficiency. On the other hand, given that a person is female (=*q*) the probability that this person being not male (not *p*) should also be large, which measures perceived necessity. In other words, this is because *p* and *q* become mutually exclusive to the extent that not *p* implies *q*. Thus, in the case that not *p* implies *q*, it is also the case that *q* implies not *p* (see Appendix 1).

It is, therefore, not necessary to reverse the antecedent and consequent clauses of conditionals and disjunctives for studying the effects of perceived sufficiency and necessity in this experiment. Because perceived sufficiency affects MP–MT responses without affecting DA–AC responses, the effect of sufficiency can be studied by observing MP–MT responses while using HS–HN items, MS–LN items, and LS–LN items (here H, M, L, S, and N stand for high, medium, low, sufficiency, and necessity, respectively.). On the other hand, because perceived necessity affects DA–AC responses without affecting MP–MT responses, the effect of necessity can be studied by observing DA–AC responses while also using HS–HN items, MS–LN items, and LS–LN items.

A preliminary study

The reduced problems, 1 for measuring perceived sufficiency and 1 for measuring perceived necessity, were constructed from each of 36 conditionals originally selected to represent high, medium, and low degrees of perceived sufficiency. For a conditional, "If a person does not own a house, then this person rents a house", the reduced problem for measuring perceived sufficiency is: Given that a person does not own a house, how probable is it that this person rents a house? The reduced problem for measuring perceived necessity is: Given that a person rents a house, how probable is it that this person does not own a house?

Twenty-nine college students served as participants in rating 36 reduced problems for measuring perceived sufficiency and 36 reduced problems for measuring perceived necessity with an 11-point scale. The final list of 12

conditionals and corresponding 12 disjunctives were used in the present experiment. The three degrees of sufficiency were adequately represented: .86 versus .57 versus .37, each pair wise difference being statistically significant. For the three degrees of necessity (.93 vs. .59 vs. .54), only the difference between the high necessity and medium necessity conditions was statistically significant.

Participants and problems

The participants were 43 freshmen enrolled in an introductory psychology course at Dong Hai University. They participated in the experiment voluntarily.

There were two types of problems: reduced and complete problems for the disjunctive reasoning task as well as for the conditional reasoning task. The reduced problems were the same for both tasks. With respect to the reduced problems, the second premise and conclusion of MP, MT, DA, and AC arguments were combined to form a reduced problem, presented in one-sentence question form, as for MP (see Table 1): "Given not *p*, how probable is *q*?" For the complete problems in the conditional reasoning task, MP, MT, DA, and AC arguments took the two-sentence form consisting of the conditional-statement premise and their respective reduced problem to obtain a complete problem. For the complete problems in the disjunctive reasoning task, MP, MT, DA, and AC comparable arguments (referred to as MP′, MT′, DA′, and AC′, respectively) also took the two-sentence form consisting of the disjunctive-statement premise and the same reduced problem to obtain a complete problem.

Design and procedure

The design was a 2 (task) × 2 (problem) × 3 (sufficiency) × 4 (argument) factorial. Task (conditional or disjunctive reasoning) was a between-subjects variable. Problem (reduced or complete problem), sufficiency (high, medium, or low), and argument (MP/MP′, etc.) were within-subjects variables.

The 43 participants were randomly assigned to one group to receive one task and another group to receive the other task. Twenty participants received the conditional reasoning task, while 23 participants received the disjunctive reasoning task.

The participants served in the experiment in large groups. They worked out two practice problems printed on the front page of a booklet before attempting to solve experimental problems. The first practice problem was in the *reduced* form, "Given that Mary is an *A* High School student, how probable is it that she is going to a picnic today?" They were to answer the problem by indicating their judged probability on an 11-point scale that ranged from 0 to 100%, with 0 standing for "absolutely improbable" and 100 for "absolutely certain". The second problem was in the *complete* form, "If Mary is an *A* High School student, then she is going to a picnic today. Given that Mary is an *A* High School student, how probable is it that she is going to a picnic today?" They were told that the first sentence stated an assumption and that they were to answer the second sentence under the assumption by indicating their judged probability on the same 11-point scale. They were reminded to write down their answer, relying on their own judgment. Then, participants rated 48 experimental problems (to be described below) at their own pace.

For about half the disjunctive reasoning group (23) and half the conditional reasoning group (20), the first 24 experimental problems were in the reduced form and the last 24 in the complete form. The order was reversed for the remaining participants. For each participant the first set of 24 experimental problems was constructed by randomly selecting two out of each set of four disjunctives (or conditionals) of different degrees of perceived sufficiency. Since each disjunctive (or conditional) could be used for constructing four types of arguments (MP, MT, DA, and AC, or comparable ones), there resulted 24 experimental problems altogether. The complementary set of six disjunctives (or conditionals) was used to construct the second set of 24 experimental problems. For each participant, one set of 24 problems was in the reduced form, while the other set of 24 problems was in the complete form. Thus, when one participant saw 24 problems in the reduced form, another participant saw these same 24 problems in the complete form. Within each set of the reduced or complete forms, there were two randomized orders and two respective reverse orders of presenting the 24 problems.

Results

Since 12 participants assigned to the disjunctive task (subgroup 1) received the reduced problems first and the complete problems second, while the order was reversed for the remaining 11 participants (subgroup 2), an ANOVA (2 subgroups × 2 types of problem × 3 degrees of sufficiency × 4 types of argument) was conducted to determine whether there was an order effect. It was found that both the effect of subgroup and the interaction between subgroup and type of problem were not significant (Fs < 1). A similar ANOVA performed for the participants assigned to the conditional task (10 for subgroup 1 and 10 for subgroup 2) showed also that both effect of subgroup and the interaction between subgroup and type of problem were not significant (Fs < 1). Therefore, the data obtained from the two subgroups were combined for each task in the following analyses.

Table 2 Mean probability ratings for the reduced and complete valid problems: conditional versus disjunctive reasoning

Perceived sufficiency	Type of problem	Conditional		Disjunctive	
		MP	MT	MP′	MT′
High	Reduced	.82	.82	.82	.82
	SD	.19	.20	.17	.18
	Complete	.93	.90	.90	.83
	SD	.10	.13	.15	.19
	Increase	.11*	.08	.08	.01
Medium	Reduced	.60	.57	.62	.62
	SD	.12	.13	.17	.18
	Complete	.84	.65	.73	.61
	SD	.12	.17	.15	.17
	Increase	.24*	.08	.11*	−.01
Low	Reduced	.43	.49	.46	.50
	SD	.13	.17	.20	.17
	Complete	.78	.60	.66	.70
	SD	.19	.13	.18	.17
	Increase	.35*	.11*	.20*	.20*

MP modus ponens, *MT* modus tollens, *MP′* comparable to MP, *MT′* comparable to MT, *DA* denial of the antecedent, *AC* affirmation of the consequent, *DA′* comparable to DA, *AC′* comparable to AC

* $p < .025$

Table 3 Mean probability ratings for the reduced and complete invalid problems: conditional versus disjunctive reasoning

Perceived necessity	Type of problem	Conditional		Disjunctive	
		DA	AC	DA′	AC′
High	Reduced	.93	.94	.86	.87
	SD	.14	.13	.18	.23
	Complete	.93	.94	.87	.86
	SD	.17	.13	.19	.20
	Increase	.00	.00	.01	−.01
Medium	Reduced	.58	.57	.60	.65
	SD	.12	.14	.18	.19
	Complete	.64	.73	.60	.66
	SD	.21	.14	.21	.22
	Increase	.06	.16*	.00	.01
Low	Reduced	.60	.59	.62	.54
	SD	.15	.14	.21	.20
	Complete	.65	.71	.62	.67
	SD	.21	.22	.27	.25
	Increase	.05	.12	.00	.13*

MP modus ponens, *MT* modus tollens, *MP′* comparable to MP, *MT′* comparable to MT, *DA* denial of the antecedent, *AC* affirmation of the consequent, *DA′* comparable to DA, *AC′* comparable to AC

* $p < .025$

The reduced problems

The reduced problems for the conditional reasoning task being identical to the reduced problems for the disjunctive reasoning task, the comparable results obtained from these two tasks are presented in Tables 2 and 3. It can be seen from these tables that the mean probability rating obtained for each condition of the conditional reasoning task is generally comparable to the mean probability rating obtained for each corresponding condition of the disjunctive reasoning task.

Three-way ANOVAs (task by argument by sufficiency or necessity) were performed separately for MP–MT or MP′–MT′ (which is omitted in the following, if there is no ambiguity from the context) and DA–AC on the mean probability ratings observed from the reduced problems. With respect to MP–MT (task by MP–MT by sufficiency), only the effect of sufficiency was significant, $F(2, 82) = 69.87$, MSE = .038, $p < .01$, $\eta^2 = .630$. With respect to DA-AC (task by DA–AC by necessity), only the effect of necessity was significant, $F(2, 82) = 75.94$, MSE = .035, $p < .01$, $\eta^2 = .649$. The simple effects of sufficiency were calculated to see whether the manipulation of sufficiency produced the following ordering of probability ratings: high sufficiency > medium sufficiency > low sufficiency. Similarly, further simple effects of necessity were calculated to see whether the manipulation of necessity produced the following ordering of probability ratings: high necessity > medium necessity = low necessity. The results are presented in Table 4. It can be seen from the table that the manipulation of both sufficiency and necessity was generally adequate in producing their effects on the probability ratings. Although the mean rating on the medium necessity condition was significantly higher than the mean rating on the low necessity condition for AC reduced problems in disjunctive reasoning, this result does not contradict the original finding that the ratings on the medium necessity items were not significantly higher than the ratings on the low necessity items for some independent group of participants. This is because a conclusion of non-significance merely represents an acceptance of a null hypothesis, which is known to be inaccurate.

Complete problems: first conditionalization as a component

The mean probability ratings observed for the reduced and complete MP–MT (or MP′–MT′) problems are presented in Table 2 as a function of task and perceived sufficiency. With the comparable ratings on the same reduced problems as the base lines for the conditional and disjunctive tasks, adjustments in the ratings from the reduced to complete problems represent assumption-based reasoning.

Table 4 Effects of sufficiency and necessity on rating the reduced problems: conditional versus disjunctive reasoning

Sufficiency		Necessity	
Conditional			
MP	High > Medium > Low	DA	High > Medium = Low
MT	High > Medium > Low	AC	High > Medium = Low
Disjunctive			
MP′	High > Medium > Low	DA′	High > Medium = Low
MT′	High > Medium > Low	AC′	High > Medium > Low

MP modus ponens, *MT* modus tollens, *MP′* comparable to MP, *MT′* comparable to MT, *DA* denial of the antecedent, *AC* affirmation of the consequent, *DA′* comparable to DA, *AC′* comparable to AC

For finding out whether the first conditionalization is a component in solving complete MP–MT problems, we inspect whether the same sufficiency effect is observable from solving complete MP–MT (or MP′–MT′) problems as from solving reduced MP–MT (or MP′–MT′) problems. This tendency seems obvious from Table 2. A three-way ANOVA (task by sufficiency by MP-MT argument) performed on rating the complete problems confirms this tendency. Thus, the sufficiency effect was significant, $F\,(2,\ 82) = 50.68$, $\mathrm{MSE} = .021$, $p < .01$, $\eta^2 = .553$. The interaction between sufficiency and task was not significant, $F < 1$, indicating that the effect of sufficiency on rating the complete problems did not differ between conditional and disjunctive tasks.

With respect to complete DA–AC (or DA′–AC′) problems, a similar tendency is observable from Table 3. A three-way ANOVA (task by necessity by DA–AC argument) performed on rating the complete problems showed that the effect of necessity was significant, $F\,(2,\ 82) = 36.28$, $MSE = .045$, $p < .01$, $\eta^2 = .469$. As in the case of MP-MT, the interaction between necessity and task was not significant ($F < 1$), indicating that the effect of necessity on rating the complete problems did not differ between conditional and disjunctive tasks.

Complete problems: effects of different tasks

It can be seen from Table 2 that different patterns of assumption-based reasoning seem observable from the two different tasks. Assumption-based reasoning observable from MP looks symmetric with respect to MT especially in the medium and low sufficiency conditions for disjunctive reasoning. On the other hand, assumption-based reasoning for MP is more pronounced than assumption-based reasoning for MT in conditional reasoning, which is a typical finding from conditional reasoning tasks. As will be seen, these patterns were produced by a decrease of assumption-based reasoning for MP′ to the level of MT′ in disjunctive reasoning.

A four-way ANOVA (task by problem by valid argument by sufficiency) supports the MP′–MT′ symmetry for disjunctive reasoning. Thus, the main effect of argument was significant, $F\,(1,\ 41) = 14.31$, $\mathrm{MSE} = .014$, $p < .01$, $\eta^2 = .259$, indicating that MP responses were generally higher than MT responses. The task by problem by argument interaction was significant, $F\,(1,\ 41) = 4.30$, $\mathrm{MSE} = .016$, $p < .05$, $\eta^2 = .095$. This interaction indicates that the finding of higher MP responses than MT responses was obtained mainly from conditional reasoning. Moreover, MP and MT responses refer to those observed from the complete problems. The task by problem by sufficiency by argument was also significant, $F\,(2,\ 82) = 4.98$, $\mathrm{MSE} = .010$, $p < .01$, $\eta^2 = .108$. This four-way interaction indicates that the finding of MP responses higher than MT responses in conditional reasoning is observed largely in the medium and low sufficiency conditions. The problem by sufficiency interaction was significant, $F\,(2,\ 82) = 9.25$, $MSE = .026$, $p < .01$, $\eta^2 = .184$. This significant interaction indicates that assumption-based reasoning tends to be observed from the low sufficiency than from the high sufficiency condition.

To see different MP–MT patterns for conditional and disjunctive reasoning more clearly, a three-way ANOVA (task by sufficiency by argument) was conducted on assumption-based reasoning measures as a dependent variable. It was found that the argument by task interaction was significant, $F\,(1,\ 41) = 4.30$, $\mathrm{MSE} = .032$, $p < .05$, $\eta^2 = .095$, indicating that assumption-based reasoning for MP and MT depended on the task. A simple effect analysis showed that the assumption-based component for MP′ (.13) did not differ significantly from the assumption-based component for MT′ (.07) in disjunctive reasoning [$t\,(22) = 1.90$, $p = .07$], although assumption-based reasoning for MP (.24) was significantly larger than assumption-based reasoning for MT (.09) in conditional reasoning [$t\,(19) = 4.42$, $p < .01$].

With respect to DA–AC responses, a four-way ANOVA (task by problem by argument by necessity) involving DA–AC as an argument variable was conducted as for MP–MT responses. It was found that the main effect of argument was not significant, $F\,(1, 41) = 2.14$, $\mathrm{MSE} = .021$, indicating that DA responses did not differ significantly from AC responses. It was further found that the effect of task and all the interactions involving task were not significant, all Fs < 1. Thus, with respect to DA and AC, disjunctive reasoning did not differ from conditional reasoning.

Discussion

Although the two major premises, "either *p* or *q*" and "if not *p* then *q*", are comparable in the material interpretation of conditionals, they look quite different in meaning.

The findings that the variables of perceived sufficiency and necessity affect conditional reasoning are well documented (e.g., Bucci, 1978, Experiment 2; Byrne, 1989; Cummins et al., 1991; Rumain et al., 1983; Markovits, 1984; Staudenmayer, 1975; Thompson, 1994, 1995). The variables of perceived sufficiency and necessity seem to be unrelated to disjunctive reasoning.

In spite of the apparent differences between disjunctive and conditional statements, it is surprising to find that the denial inferences constructed from the former and the MP–MT inferences constructed from the latter are equally affected by perceived sufficiency. It is also surprising to find that the affirmation inferences and the DA–AC inferences are equally affected by perceived necessity. In other words, P (conclusion | second premise) is a significant component in the denial inferences and affirmation inferences as it is a significant component in MP–MT and DA–AC responses.

In the present experiment, sufficiency and necessity may appear to be almost perfectly confounded by being manipulated by one and the same factor. As was pointed out earlier, however, it was impossible to find high sufficiency conditional relationships that are characterized by low necessity in the present case. On the other hand, it is well documented that MP–MT responses are affected by perceived sufficiency without being affected by perceived necessity, while DA–AC responses are affected by perceived necessity without being affected by perceived sufficiency. Thus, not only are different dependent variables or different data (MP–MT responses vs. DA–AC responses) are involved, but also the effective independent variables are different for these two types of responses. These facts will become transparent in the discussion of Experiment 2. As a result, there could not be a confounding in the manipulation of variables.

As was observed in previous studies (e.g., Chou, 2009; Klauer et al., 2010; Liu, 2003; Liu et al., 1996; Matarazzo & Baldassarre, 2008; Schroyens, 2009), the discrepancies in MP responses, which stand for the assumption-based reasoning, for solving problems with and without rules are larger in low sufficiency than in high sufficiency conditions. This is because reasoning responses in solving MP problems without rules are already high in the high sufficiency condition and an upward increase in reasoning responses by adding the rules could not be as high as in the low sufficiency condition due to the ceiling effect. The same tendency is also observable for MP′. Similar findings are observable in the MT cases.

It was also observed that differences between disjunctive and conditional reasoning appear when P (conclusion | second premise) is computed under the assumption of different major premises in support of the successive-conditionalization approach. That the two denial inferences are more symmetric than the MP and MT inferences was found to be due to the second conditionalization. This finding demonstrates that reasoners still take the meaning of "either p or q" to be somewhat different from the meaning of "if not p then q", although both are comparable in the material interpretation of conditionals. The level of MP responses in conditional reasoning higher than that of MP′ in disjunctive reasoning is also in line with the successive-conditionalization approach. The advantage of the present approach over other approaches, such as mental model theory (Johnson-Laird et al., 1992), is that it is possible to locate the origin of the superiority of conditional reasoning to disjunctive reasoning precisely in assumption-based reasoning.

Because the purpose of the experiment was to test the material interpretation of conditionals, whether any difference between disjunctive and conditional reasoning is due to the negative form of conditionals deserves our consideration. There are several studies (e.g., Evans, 1977; Pollard & Evans, 1980; Wildman & Fletcher, 1977) comparing the percentages of conditional inferences on negated rules (including "if p then q" and "if not p then q"). The general findings are: (a) MP inferences are nearly identical, and (b) all inferences are made more often on rules where the conclusion stated is negative rather than affirmative, which is known as negative conclusion bias. These results could be due to the fact that early studies used abstract materials. Although the conditional statement is in the form, if not p then q, in Experiment 1, MT inferences (affirmative conclusions) seem not particularly affected. Thus, the conclusion of Experiment 1 seems not due to the negative form of "if not p then q".

It is well documented that a biconditional pattern (endorsing both DA, and AC) is also observed in conditional reasoning (e.g., Evans et al., 1993). Similarly, with respect to the inclusive or exclusive interpretation of disjunctives, Braine and Rumain (1981) found 41% exclusive truth tables as opposed to 32% inclusive. As an advantage of the present approach, it is possible to find whether participants adopted an exclusive reading when participants from the same subject pool adopted a biconditional reading. This can easily be assessed by inspecting the presence of assumption-based reasoning in DA–AC (DA′–AC′), because the presence of assumption-based reasoning represents a biconditional (or exclusive) reading.

As is clear from Tables 2 and 3, a biconditional pattern is not generally observed for DA–AC when a conditional pattern is generally observed for MP–MT in the low and medium sufficiency/necessity conditions for the conditional reasoning group. In parallel to this finding, an exclusive reading is not generally observed for DA′–AC′ when an inclusive reading is generally observed for MP′–MT′ in the low and medium sufficiency/necessity conditions for the disjunctive reasoning group. In the high sufficiency/necessity condition, it is often difficult to assess whether participants adopt a conditional (inclusive) or biconditional (exclusive) reading of conditionals (disjunctives) because of the ceiling effect.

Remark 1

In the material interpretation of conditionals, "If *p* then *q*" is to be equated with "Either not *p* or *q*", the reason we did not use this contrast but we used the contrast of "If not *p* then *q*" versus "Either *p* or *q*" in testing the material interpretation hypothesis is as follows. In everyday language, two disjuncts in a disjunctive statement should be two alternatives. One disjunct should not include another. However, not *p* is likely to include *q*, because *p* is generally different from *q*. A simple example suffices to illustrate how difficult "Either it is not a dog or it is an animal" is to comprehend. On the other hand, its contrasting conditional, "If it is a dog, then it is an animal" (high sufficiency, low necessity item), is easy to comprehend. It is, therefore, not a surprise to find that correct responses for the denial and affirmation inferences in the NA cases (1st disjunct negative, 2nd disjunct affirmative) are generally below 50%, while correct responses in the AA cases are generally above 80% in previous studies (Evans et al., 1993, p. 148). It is therefore not feasible to test the material interpretation hypothesis by contrasting "If *p* then *q*" versus "Either not *p* or *q*".

Remark 2

The question arises as to why disjunctive reasoning is affected by perceived sufficiency or necessity, because disjunctives seem quite symmetric. We first note that P (*p* therefore *q*), which is equal to P ($q \mid p$), does not always stand for perceived sufficiency. Suppose the major premise is "If *q* then *p*". Then P ($q \mid p$) stands for perceived necessity, and P ($q \mid p$, if *q* then *p*) for AC. Thus, whether P ($q \mid p$) stands for perceived sufficiency or necessity depends on whether *p*–*q* refer to the antecedent–consequent or the consequent–antecedent of the conditional-statement premise. In other words, whether P ($q \mid p$) stands for perceived sufficiency or necessity depends on how perceived sufficiency or necessity is inherited from the comparable conditional argument form in testing the material interpretation hypothesis. The true picture is that perceived sufficiency or necessity is definable by the first conditionalization involved in denial inferences or in affirmation inferences also in disjunctive reasoning. In other words, perceived sufficiency or necessity is an intrinsic variable that affects disjunctive reasoning as it affects conditional reasoning.

Experiment 2: syllogistic versus conditional reasoning

As in conditional and disjunctive argument forms that consist of two premises and a conclusion, all syllogisms also have two premises and a conclusion. A similar successive-conditionalization routine, therefore, applies to syllogistic reasoning as to disjunctive reasoning. Because the major premise of each conditional argument form is a conditional statement, it can be translated into a universal statement in such a way that the four conditional argument forms become four types of syllogisms as presented in Table 5.

Table 5 Conditional versus comparable syllogistic argument forms

Conditional		Syllogistic	
MP:	If X is B then X is C X is B, therefore X is C	MP″:	All B are C X is B, therefore X is C
MT:	If X is B then X is C X is not C, therefore X is not B	MT″:	All B are C X is not C, therefore X is not B
DA:	If X is B then X is C X is not B, therefore X is not C	DA″:	All B are C X is not B, therefore X is not C
AC:	If X is B then X is C X is C, therefore X is B	AC″:	All B are C X is C, therefore X is B

MP modus ponens, *MT* modus tollens, *MP′* comparable to MP, *MT′* comparable to MT, *DA* denial of the antecedent, *AC* affirmation of the consequent, *DA′* comparable to DA, *AC′* comparable to AC

For a universal statement (All B are C) referring to empirical events B and C, the existential import is generally satisfied. Therefore, it is logically equivalent to a conditional statement (If X is B then X is C). As in conditional reasoning, only those syllogisms comparable to MP and MT (i.e., MP″ and MT″, respectively) are valid. Thus, it is understandable from Table 5 that early investigators (e.g., Osherson, 1974; Revlin & Leirer, 1978; Revlis, 1975) put forth the hypothesis that similar processes underlie reasoning with conditional and syllogistic arguments. Guyote and Sternberg (1981) tested this hypothesis in an experiment, and found that these two types of reasoning are highly correlated.

The main purpose of Experiment 2 was then to test whether conditional reasoning is identical to syllogistic reasoning, when the conditional-statement premise is replaced by its logically equivalent universal statement in the successive conditionalization framework. The present approach predicts that assumption-based reasoning should differ between conditional and syllogistic reasoning, although the first conditionalization should be a significant component of the second conditionalization in both conditional and syllogistic reasoning.

The prediction that perceived sufficiency affects syllogistic reasoning as well as conditional reasoning differs from that of Chater and Oaksford (1999). They argued that "all B are C" should be interpreted as P (C | B) = 1. In abiding with Ramsey test, therefore, they would predict that MP″ should be rated higher than MP responses. In the present approach, participants are instructed to assume the premises to be true. These participants should, therefore, interpret B to be sufficient for C in conditional reasoning as in syllogistic

reasoning, although many participants could not detach from reality in computing the second conditionalization in both types of reasoning.

Because differences in the surface features of the major premises (conditional vs. universal statements) affect the second conditionalization without affecting the first conditionalization, the present approach makes a different prediction from the logical-equivalence hypothesis. The latter predicts that the logically equivalent premises entail identical reasoning. It is known that, in solving syllogisms, however, many people may rely on spatial or geometric intuitions. Thus, there are theories of syllogisms based on Euler circles or Venn diagrams (e.g., Erickson, 1974). More recently, Ford (1994) identified two main groups of participants in solving syllogisms. One group represented the relationship between classes mainly in a spatial manner. The other group used a primarily verbal representation. No such spatial representation is reported in solving conditional arguments.

Solving MT problems is known to be more difficult than solving MP problems in conditional reasoning. In the successive-conditionalization framework, this is because it is difficult to compare "given not C, therefore not B" to "if B then C" in the second conditionalization. The difficulty arises because both are not only opposite in direction, but also the former have negative signs. In solving MT″ problems, participants attempt to compare "given that X is not C, therefore X is not B" to "all B are C" in the second conditionalization. If many participants understand "All B are C" by representing this premise spatially as "B is included in C", then the problem of opposite direction seems alleviated in comparing "given that X is not C, therefore X is not B" to "all B are C". This is because, assuming that B is included in C, something outside of C is certainly outside of B. It is, therefore, predicted that MT″ are not very difficult as compared to MP″, although the former are still more difficult than the latter are. In other words, MT″ should be easier than MT problems in conditional reasoning.

Method

Participants and problem materials

The participants were 134 freshmen enrolled in an introductory psychology course at Da-Yeh University, a small university in the central part of Taiwan. They participated in the experiment for a partial fulfillment of the course requirement.

All the problems used in the present experiment were generated from 12 conditionals adopted from previous studies (Liu, 2003; Liu et al., 1996): 4 conditionals of high sufficiency and low necessity, 4 conditionals of medium sufficiency and low necessity, and 4 conditionals of low sufficiency and low necessity (see Appendix 2). These 12 conditionals were used to generate 12 reduced problems and 12 complete problems in the forward conditional condition. The 12 conditionals were written into 12 universal statements to generate 12 reduced problems and 12 complete problems in the forward syllogistic condition. As with Cummins (1995), Thompson (1995), and Weidenfeld, Oberauer, and Hornig (2005), the conditional relationships ("if cause then effect", "if category then property", "if member then category") are referred to as forward, while the reversed conditional relationships are referred to as backward.

For manipulating perceived necessity as well as perceived sufficiency, the antecedent and consequent clauses of the original 12 conditionals were reversed to obtain 12 backward conditionals: 4 conditionals of low sufficiency and high necessity; 4 conditionals of low sufficiency and medium necessity, and 4 conditionals of low sufficiency and low necessity. The 12 reversed conditionals were used to generate 12 reduced problems and 12 complete problems in the reversed conditional condition. The 12 reversed conditionals were also written into 12 reversed universal statements to generate 12 reduced problems and 12 complete problems in the reversed syllogistic condition.

Tasks and procedure

Half the participants (67) were randomly assigned to the two forward conditions: 34 to the forward conditional condition and 33 to the forward syllogistic condition. The remaining participants (67) were randomly assigned to the two reversed conditions: 34 to the reversed conditional condition and 33 to the reversed syllogistic condition. About half the participants in each subgroup of 33 or 34 received the reduced problems first and the complete problems next, while the order was reversed for the remaining participants as in Experiment 1. All the other details of the experimental procedure were the same as in Experiment 1.

Results

There were four groups in the present experiment: one group in the forward conditional condition, one group in the forward syllogistic condition, one group in the reversed conditional condition, and one group in the reversed syllogistic condition. Although about half the participants received the reduced problems first and the complete problems next, while the order was reversed for the remaining participants in each group, ANOVAs showed that the order effect was not significant for each pair of groups assigned to the conditional as well as the syllogistic condition. Therefore, the data obtained from the two groups were combined for each condition in the following analyses.

Table 6 Forward version—mean probability ratings for conditional versus syllogistic reasoning: effects of sufficiency

Perceived sufficiency	Kind of problem	Type of argument							
		C	S	C	S	C	S	C	S
		MP	MP″	MT	MT″	DA	DA″	AC	AC″
High	Reduced	.91	.93	.74	.83	.56	.55	.49	.48
	SD	.14	.11	.20	.19	.12	.18	.16	.21
	Complete	.95	.98	.76	.81	.58	.58	.55	.59
	SD	.11	.04	.21	.19	.19	.15	.26	.15
	Increase	.04*	.05*	.01	−.03	.02	.03	.07	.11*
Medium	Reduced	.72	.72	.61	.62	.68	.67	.60	.60
	SD	.13	.14	.14	.17	.15	.16	.15	.15
	Complete	.88	.92	.76	.77	.64	.71	.64	.66
	SD	.15	.08	.19	.17	.26	.18	.21	.16
	Increase	.16*	.20*	.14*	.15*	−.03	.04	.04	.06
Low	Reduced	.53	.51	.50	.50	.54	.53	.64	.60
	SD	.15	.15	.12	.18	.13	.18	.13	.18
	Complete	.74	.80	.59	.67	.55	.62	.64	.73
	SD	.25	.18	.26	.16	.21	.14	.23	.16
	Increase	.21*	.29*	.09	.17*	.01	.09*	.00	.13*

MP modus ponens, *MT* modus tollens, *MP′* comparable to MP, *MT′* comparable to MT, *DA* denial of the antecedent, *AC* affirmation of the consequent, *DA′* comparable to DA, *AC′* comparable to AC, *C* conditional, *S* syllogistic

* $p < .025$

First conditionalization

The mean probability ratings obtained in the forward conditional and syllogistic conditions are presented in Table 6 as a function of task, sufficiency, argument, and problem. On the other hand, the mean probability ratings obtained in the reversed conditional and syllogistic conditions are presented in Table 7 as a function of task, necessity, argument, and problem. There are two main findings that are readily seen from these two tables. First, it can be seen from Table 6 that both ratings on the MP–MT (including MP″–MT″, which will be omitted subsequently, when there is no ambiguity from the context) reduced and complete problems increased as a function of sufficiency, while no such tendency is observable from DA–AC. It can be seen also from Table 7 that both ratings on the DA–AC reduced and complete problems increased as a function of necessity, while no such tendency is observable from MP–MT. Second, increases in the mean ratings from the reduced to complete problems are observable from each sufficiency condition for MT″ in the forward syllogistic condition, while no such tendency is observable in the forward conditional condition. Similarly, increases in the mean ratings from the reduced to complete problems are observable from each necessity condition for MT″ in the reversed syllogistic condition, while no such tendency is observable in the reversed conditional condition.

Three-way ANOVAs were performed on ratings on the reduced problems separately for MP–MT (task by argument by sufficiency) in the forward conditions and DA–AC (task by argument by necessity) in the reversed conditions as follows. With respect to MP–MT, the effect of task was not significant, $F < 1$. The interactions involving task were all not significant. The sufficiency effect was significant, $F\,(2, 130) = 137.65$, MSE = .029, $p < .01$, $\eta^2 = .679$. These results indicate that the first conditionalization is affected by sufficiency for MP–MT in solving reduced syllogistic problems as well as in solving reduced conditional problems. Type of argument had a significant effect, $F\,(1, 65) = 64.46$, MSE = .013, $p < .01$, $\eta^2 = .486$. The argument by sufficiency interaction was also significant, $F\,(2, 130) = 6.49$, MSE = .017, $p < .01$, $\eta^2 = .091$, indicating that the effect of sufficiency is more pronounced in the high sufficiency condition than in the lower sufficiency conditions.

For another ANOVA performed on ratings on the reduced problems for DA–AC in the reversed conditions, the necessity effect was significant, $F\,(2, 130) = 133.35$, MSE = .025, $p < .01$, $\eta^2 = .672$. The effect of task was not significant, $F < 1$. The interactions involving task were all not significant. These results again indicate that the first conditionalization is affected by necessity for DA–AC in reduced syllogistic problems as well as in solving reduced conditional problems. Type of argument had a significant effect, $F\,(1, 65) = 81.66$, MSE = .017, $p < .01$, $\eta^2 = .557$. The argument by necessity interaction was also significant, $F\,(2, 130) = 5.19$, MSE = .020, $p < .01$, $\eta^2 = .074$, indicating that the effect of necessity is more pronounced in the high necessity condition than in the lower necessity conditions.

Table 7 Reverse version—mean probability ratings for conditional versus syllogistic reasoning: effects of necessity

Perceived necessity	Kind of problem	Type of argument							
		C	S	C	S	C	S	C	S
		MP	MP″	MT	MT″	DA	DA″	AC	AC″
High	Reduced	.47	.50	.51	.52	.75	.75	.90	.92
	SD	.14	.16	.13	.16	.20	.26	.15	.14
	Complete	.79	.82	.62	.68	.72	.78	.93	.94
	SD	.17	.19	.18	.25	.26	.21	.12	.12
	Increase	.32*	.32*	.11*	.17*	−.03	.04	.03	.02
Medium	Reduced	.54	.59	.63	.64	.61	.56	.72	.73
	SD	.14	.15	.20	.16	.14	.17	.13	.15
	Complete	.84	.86	.57	.76	.72	.74	.82	.75
	SD	.14	.18	.27	.22	.21	.18	.14	.22
	Increase	.30*	.27*	−.06	.12*	.11*	.17*	.10*	.02
Low	Reduced	.62	.59	.54	.51	.49	.48	.53	.54
	SD	.14	.14	.12	.14	.18	.16	.12	.10
	Complete	.83	.84	.63	.66	.57	.63	.71	.68
	SD	.16	.16	.19	.23	.14	.21	.19	.21
	Increase	.21*	.25*	.09*	.15*	.08*	.15*	.17*	.14*

MP modus ponens, *MT* modus tollens *MP′* comparable to MP, *MT′* comparable to MT, *DA* denial of the antecedent, *AC* affirmation of the consequent, *DA′* comparable to DA, *AC′* comparable to AC, *C* conditional, *S* syllogistic

* $p < .025$

Second conditionalization

In order to see whether the first conditionalization is an important component in solving complete syllogistic problems, we performed ANOVAs to find whether sufficiency (necessity) affected the complete MP–MT problems (complete DA–AC problems) in the forward syllogistic condition (reversed syllogistic condition) as in the forward conditional condition (reversed conditional condition). A three-way ANOVA (task by sufficiency by valid argument) performed on the mean ratings on the complete problems in the forward conditional and syllogistic conditions showed that the sufficiency effect was a significant source of variance: $F\,(2,\ 130) = 48.68$, MSE = .022, $p < .001$, $\eta^2 = .428$. The interaction between sufficiency and task was not significant, $F < 1$, indicating that the effect of sufficiency was observed to the same extent in solving complete syllogistic problems as in solving complete conditional problems. Another three-way ANOVA (task by necessity by invalid argument) performed on the mean ratings on the complete problems in the reversed conditional and syllogistic conditions showed that necessity was a significant source of variance: $F\,(2,\ 130) = 51.24$, MSE = .025, $p < .01$, $\eta^2 = .441$. The interaction between necessity and task was not significant, $F\,(2,\ 130) = 1.31$, MSE = .025, indicating that the effect of necessity was observed to the same extent in solving complete syllogistic problems as in solving complete conditional problems.

To find whether the second conditionalization involved in solving complete syllogistic problems differs from the second conditionalization involved in solving complete conditional problems, assumption-based reasoning was computed for syllogistic and conditional tasks under various conditions. An increase in the mean rating from the reduced to complete problems for each type of argument represents the presence of assumption-based reasoning. Table 6 (Table 7) presents increases in the mean ratings in three rows, each row for one condition of perceived sufficiency (necessity). Minus signs indicate decreases in the mean ratings.

A four-way ANOVA (task by problem by valid argument by sufficiency) was performed on the mean ratings observed from both conditional and syllogistic tasks under all the sufficiency conditions (forward version). The interaction between valid argument and problem was significant, $F\,(1,\ 65) = 13.84$, MSE = .016, $p < .01$, $\eta^2 = .176$. Thus, a standard finding of more marked assumption-based reasoning observed from MP than from MT in conditional reasoning is also observed in syllogistic reasoning. However, the valid argument by problem by task was not significant, $F < 1$, indicating that the assumption-based reasoning involved in MT″ (mean = .10) did not differ significantly from the assumption-based reasoning involved in MT (mean = .08). Because the assumption-based reasoning involved in MT and MT″ could be concealed, especially, in the high sufficiency condition due to the ceiling effect, a planned test was conducted as follows.

To test the significance of the presence of assumption-based reasoning, an ANOVA (2 tasks × 2 types of problem) was performed for each degree of sufficiency for each

type of argument. A planned test was then conducted to assess the simple effect of problem type (reduced vs. complete problem type) for each task by applying a Bonferroni adjustment ($p < .05/2$). As is apparent from Table 6, assumption-based reasoning was observed for MT″ in the low sufficiency condition for syllogistic reasoning but not for MT in conditional reasoning task.

A four-way ANOVA (task by problem by valid argument by necessity) was performed on the mean ratings observed from both conditional and syllogistic tasks under all the necessity conditions (reverse version). The effect of task was not significant. The interaction between valid argument and problem was significant, F (1, 65) = 76.74, MSE = .022, $p < .01$, $\eta^2 = .541$. A standard finding of more marked assumption-based reasoning observed from MP than from MT in conditional reasoning is also observed in syllogistic reasoning. The interaction of task by problem by valid argument was also significant: F (1, 65) = 5.85, MSE = .022, $p < .05$, $\eta^2 = .083$. This significant interaction indicates that assumption-based reasoning for MT″ (mean = .15) was more pronounced in syllogistic reasoning than for MT (.05) in conditional reasoning. The only other significant interaction involving task was obtained between task and necessity: F (2, 130) = 3.23, MSE = .025, $p < .05$, $\eta^2 = .047$. This significant interaction indicates that the significantly higher MT responses obtained for syllogistic reasoning than for conditional reasoning were observed mainly in the medium necessity condition (see Table 7).

With respect to DA and AC, a four-way ANOVA (task by problem by invalid argument by sufficiency) was performed on the mean ratings observed from both conditional and syllogistic tasks under all the sufficiency conditions (forward version). The effect of task and all the interactions involving task were not significant, indicating that DA and AC responses did not differ between syllogistic and conditional reasoning. Another four-way ANOVA (task by problem by invalid argument by necessity) was performed on the mean ratings observed from both conditional and syllogistic tasks under all the necessity conditions (reverse version). The effect of task was not significant. The only significant interaction involving task was the task by problem by invalid argument interaction: F (1, 65) = 6.97, MSE = .022, $p < .05$, $\eta^2 = .097$. This significant interaction seems to indicate that probability ratings increased from the reduced to complete DA problems more in syllogistic than in conditional reasoning. An inspection of Table 7, however, showed that this interaction is apparently due to particularly low ratings on the reduced DA problems in the medium necessity condition in syllogistic ratings, indicating that the significant interaction could be caused by a chance fluctuation in the ratings of reduced DA problems.

Discussion

The manipulation of sufficiency (or necessity) while controlling necessity (or sufficiency) was accomplished by reversing the antecedent and consequent clauses of the original 12 conditionals in Experiment 2. The effectiveness of this manipulation could be verified by examining Tables 6 and 7. It should be noted that a pair of the three means of reduced MP/MP″ problems in Table 6 (but a pair of the three means of reduced AC/AC″ problems in Table 7) gives the three corresponding means of perceived sufficiency (or necessity) actually measured from these two groups. It can be seen that the manipulation (high, medium, and low) is fairly adequate, two adjacent means being about .20 apart in the case of perceived sufficiency (.53, .72, .91; or .51, .72, .93) as well as in the case of perceived necessity (.53, .72, .90; or .54, .73, .92).

On the other hand, the control of necessity at low levels (or sufficiency at low levels) in manipulating sufficiency (or necessity) is also fairly adequate (in the range of .40–.60), although not very satisfactory (in the range of .45–.55). This is inevitable, because different samples of participants are involved. More specifically, the necessity levels measured from the two forward version groups are about .10 lower in the high sufficiency condition than in the medium and low sufficiency conditions in manipulating sufficiency (see reduced AC/AC″ problems in Table 6). Similarly, the sufficiency levels measured from the two reverse version groups are about .10 lower in the high necessity condition than in the medium and low necessity conditions in manipulating necessity (see reduced MP/MP″ problems in Table 7). As is clear from Tables 6 and 7, the means observed from both reduced and complete MP–MT problems in the high sufficiency condition are the highest, in spite of the fact that the controlled necessity levels are slightly lower in the high sufficiency condition than in the medium and low sufficiency conditions. Similarly, the means observed from both reduced and complete AC–DA problems in the high necessity condition are the highest, in spite of the fact that the controlled sufficiency levels are slightly lower in the high necessity condition than in the medium and low necessity conditions. These observations provide additional evidence that MP–MT responses are affected by perceived sufficiency without being affected by perceived necessity, while DA–AC responses are affected by perceived necessity without being affected by perceived sufficiency.

A universal statement, “all B are C”, is quite different in appearance from a conditional statement, “if X is B then X is C”, although these two statements are logically equivalent. A conditional statement can be used to generate the four argument forms, MP, MT, DA, and AC. The universal

statement that is logically equivalent to a conditional statement can also generate four syllogisms that are comparable to the four argument forms of conditional reasoning.

The two syllogisms, MP″ and MT″, comparable to MP and MT were found to be affected by perceived sufficiency equally as are the latter. The other two syllogisms, DA″ and AC″, comparable to DA and AC were found to be affected by perceived necessity equally as are the latter. In other words, the first conditionalization is a significant component in the four generated syllogisms.

It was also observed that differences between syllogistic and conditional reasoning appear in the second conditionalization. That MT″ is easier than MT was found to be due to the assumption-based reasoning. This finding is in line with the successive-conditionalization approach that reasoners still take the meaning of a universal statement, "all B are C", to be somewhat different from the meaning of a conditional statement, "if X is B, then X is C", although both are logically equivalent. Another finding in connection with the assumption-based reasoning is that MP″ was found to be only as easy as MP in conditional reasoning.

The finding that MT″ is easier than MT needs special remarks. As was pointed out earlier, an upward increase in MP–MT responses from the reduced to the complete problems may not be detectable in the high sufficiency condition due to the ceiling effect. On the other hand, such an upward increase (that represents the assumption-based component) should be detectable in the low sufficiency condition, if it is present. Thus, when sufficiency was manipulated from low to medium to high in the forward version, the finding that the assumption-based component was significantly higher for MT″ than for MT presents evidence that the former are generally easier than the latter. When necessity was manipulated from low to medium to high in the reverse version, perceived sufficiency was kept low in the three conditions. In this case, it was noted that assumption-based reasoning was more pronounced for MT″ than for MT.

Finally, as is apparent from comparing Table 6 to Table 7, DA–AC as well as DA″–AC″ responses are more often endorsed in the reverse version than in the forward version. Perhaps, presentation of a reverse version could tend to remind the forward version for the participants. It is, therefore, understandable that a biconditional pattern of conditional reasoning is more frequent in the reverse version than in the forward version. In parallel to the biconditional reading of conditionals is Dickstein's claim (1981) that participants assume that both the original (All C are B) and the converted form (All B are C) are true. In other words, this "conversion-by-addition" thesis is more common in the reverse version than in the forward version for syllogistic reasoning.

General discussion

The material interpretation of conditionals is still in dispute (e.g., Evans et al., 2005). We used this interpretation of conditionals to generate comparable disjunctive and conditional statements for studying both disjunctive and conditional reasoning processes. We included low, medium, and high sufficiency conditionals as well as their comparable disjunctives. To some extent, our low sufficiency conditionals could be comparable to the extremely improbable conditional, "If the car's battery is dead then it will start", used by Evans et al. to produce paradoxes of claiming that ordinary conditionals are material conditionals.

Experiment 1 demonstrated that both perceived sufficiency and necessity affect P (conclusion | second premise), which is a significant component in disjunctive reasoning as it is in conditional reasoning. The most interesting finding was that the easiest argument form, MP, in conditional reasoning becomes as difficult as the most difficult argument form, MT, simply by replacing the conditional-statement premise with the comparable disjunctive statement. Further analyses, however, show that this result was produced by a small increase in MP-comparable denial-inference responses in disjunctive reasoning as compared to a large increase in MP responses in conditional reasoning when reasoners compute P (conclusion | second premise) under the assumption of different major premises (i.e., second conditionalization), in line with the successive-conditionalization approach.

As is reviewed in Evans et al. (1993), if at least one of the disjuncts is negated, the resulting disjunctive reasoning is generally found to be more difficult than when both disjuncts are affirmative. As a matter of fact, when one of the disjuncts is negated, the obtained disjunctive statement could be anomalous. In Experiment 1, both disjuncts of each disjunctive statement were affirmatives. In this case, as was observed, comparable results were generally obtained for disjunctive and conditional reasoning processes, although the effect of a difference in the major premises was obtained, indicating some differential psychological effects produced by the material implication of conditionals.

A reviewer of this manuscript pointed out that "either not p or q" should be used to test the material interpretation of "if p then q", when p and q represent a cause-effect relationship. However, whether "either not p or q" is comprehensible in this case remains to be seen.

In Experiment 2, in spite of the fact that the subject term of a universal statement sounds maximally sufficient for the predicate term and the latter maximally necessary for the former (Chater & Oaksford, 1999), it was found that both perceived sufficiency and necessity equally affect the syllogistic reasoning generated from those universal statements comparable to the conditional statements of various degrees of perceived sufficiency and necessity. Furthermore, the

assumption-based reasoning of MT-comparable syllogisms was found to be easier than the assumption-based reasoning of MT problems in conditional reasoning. This finding also supports a well-known Gestalt thesis that a difficult problem becomes suddenly solvable once reasoners are able to perceive the problem situation differently.

Finally, with respect to the material interpretation of conditionals, it is generally applicable to the present study, although there are some differences in the psychological meanings of "either *p* or *q*" and "if not *p* then *q*", as reflected in the differences in assumption-based reasoning. Similarly, conditional reasoning and its comparable syllogistic reasoning are affected by perceived sufficiency and necessity in the same way, although there are some differences in the psychological meanings of "all B are C" and "if X is B then X is C", as also reflected in the differences in assumption-based reasoning.

Conclusion

The successive-conditionalization approach has advantages and limitations. As an advantage of this approach, it is possible to locate the origins of knowledge-based and form-based reasoning through an experimental procedure. In the present study, both disjunctive and syllogistic reasoning have major premises (forms) that differ from the conditional-statement premise, while both have the same second premise and the same conclusion. Thus, if disjunctive or syllogistic reasoning performance differs from conditional reasoning performance, its origin could be located in the second but not in the first conditionalization. Any other theory could make the same predictions, but only through parameter estimations.

The present approach has also limitations. Once the origin of form-based reasoning is located in the second but not in the first conditionalization, it is only possible to infer the mechanism in the differences in the major premises. The precise mechanism responsible for the differences needs further experimentation.

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Appendix 1: Disjunctives and conditionals (in Parentheses) used in Experiment 1 (Translated from Chinese)

High perceived sufficiency

H1 is a man or woman. (If H1 is not a man, then H1 is a woman.)
H2 wins or looses in a championship game. (In a championship game, if H2 does not win, then H2 looses.
It is now daytime or night time. (If it is not daytime now, it is night time.)
H4 wears the left-hand glove first or the right-hand glove first. (If H4 does not wear the left-hand glove first, then H4 wears the right-hand glove first.

Medium perceived sufficiency

M1 moves to a new house or uses old furniture. (If M1 does not move to a new house, then M1 uses old furniture.)
M2 drinks or M2 is healthy. (If M2 does not drink, then M2 is healthy.)
M3 eats lunch or is on a diet. (If M3 does not eat lunch, then M3 is on a diet.)
Something is on sale or expensive. (If something is not on sale, then it is expensive.)

Low perceived sufficiency

A woman's hair is long or lively. (If a woman's hair is not long, then she is lively.)
Somebody has a job or is sick. (If somebody does not have a job, then this person is sick.)
Somebody wears a suite or goes to a bank. (If somebody does not wear a suite, then this person goes to a bank.)
Somebody eats a stake or beef noodle. (If somebody does not eat a stake, then this person eats beef noodle.)

Note. H1 or M2 stands for a person's name.

Appendix 2: Universal statements and conditional statements (in parentheses) used in Experiment 2 (Translated from Chinese)

High perceived sufficiency

All persons living in Sweden are living in the northern hemisphere. (If a person lives in Sweden, then this person lives in the northern hemisphere.)
All diamonds are hard. (If this substance is a diamond, then it is hard.)
All five-year-olds are children. (If this person is a five-year-old, then this person is a child.)
All dogs are animals. (If it is a dog, then it is an animal.)

Medium perceived sufficiency

All persons who move to a new house add new furniture. (If a person moves to a new house, then this person adds new furniture.)
All persons who catch cold take one-day leave from the company. (If a person catches cold, then this person takes one-day leave from the company.)

All students who cheat in the exam will be punished by the school. (If a student cheats in the exam, then this student will be punished by the school.)
All husbands who come back home late will be scolded by their wife. (If the husband comes back home late, then he will be scolded by his wife.)

Low perceived sufficiency

All women who have long hair are quiet. (If a woman has long hair, then she is quiet.)
All persons who wear glasses are intelligent. (If a person wears glasses, then this person is intelligent.
All students who have good memory study in the college of liberal arts. (If a student has good memory, then this student studies in the college of liberal arts.)
All persons who put white sport shoes on go to play ping-pong. (If a person puts white sport shoes on, then this person goes to play ping-pong.)

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